# Topological Analysis of Inertial Dynamics – Supplemental Material

Antoni Sagristà, Stefan Jordan, Andreas Just, Fabio Dias, Luis Gustavo Nonato, and Filip Sadlo

This document provides supplemental material for our paper "Topological Analysis of Inertial Dynamics". Section 1 describes the numerical integration scheme.

## 1 Integration Scheme

This description of the integration scheme provides additional details to Section 3.1 of our paper. We solve

$$\mathbf{x}(t) = \mathbf{x}_0 + \int_{t_0}^{t} \dot{\mathbf{x}}(\tau)\,\mathrm{d}\tau \tag{1}$$

(Equation 5 in our paper) and concurrently

$$\dot{\mathbf{x}}(t) = \dot{\mathbf{x}}_0 + \int_{t_0}^{t} \mathbf{a}(\mathbf{x}(\tau),\tau)\,\mathrm{d}\tau, \tag{2}$$

(Equation 6 in our paper) for time $t$ and time step $\Delta t$ by coupled fourth-order Runge-Kutta (RK4) integration:

$$\mathbf{x}_1 = \mathbf{x}(t), \quad \dot{\mathbf{x}}_1 = \dot{\mathbf{x}}(t), \quad \mathbf{a}_1 = \mathbf{a}(\mathbf{x}_1, t), \tag{3}$$

$$\mathbf{x}_2 = \mathbf{x}_1 + \dot{\mathbf{x}}_1 \Delta t/2, \quad \dot{\mathbf{x}}_2 = \dot{\mathbf{x}}_1 + \mathbf{a}_1 \Delta t/2, \quad \mathbf{a}_2 = \mathbf{a}(\mathbf{x}_2, t + \Delta t/2), \tag{4}$$

$$\mathbf{x}_3 = \mathbf{x}_1 + \dot{\mathbf{x}}_2 \Delta t/2, \quad \dot{\mathbf{x}}_3 = \dot{\mathbf{x}}_1 + \mathbf{a}_2 \Delta t/2, \quad \mathbf{a}_3 = \mathbf{a}(\mathbf{x}_3, t + \Delta t/2), \tag{5}$$

$$\mathbf{x}_4 = \mathbf{x}_1 + \dot{\mathbf{x}}_3 \Delta t, \quad \dot{\mathbf{x}}_4 = \dot{\mathbf{x}}_1 + \mathbf{a}_3 \Delta t, \quad \mathbf{a}_4 = \mathbf{a}(\mathbf{x}_4, t + \Delta t). \tag{6}$$

From this, we obtain the new position $\mathbf{x}(t+\Delta t)$ and velocity $\dot{\mathbf{x}}(t+\Delta t)$ from the previous $\mathbf{x}(t)$ and $\dot{\mathbf{x}}(t)$ as follows:

$$\mathbf{x}(t+\Delta t) = \mathbf{x}_1 + (\dot{\mathbf{x}}_1 + 2\dot{\mathbf{x}}_2 + 2\dot{\mathbf{x}}_3 + \dot{\mathbf{x}}_4)\Delta t/6, \tag{7}$$

$$\dot{\mathbf{x}}(t+\Delta t) = \dot{\mathbf{x}}_1 + (\mathbf{a}_1 + 2\mathbf{a}_2 + 2\mathbf{a}_3 + \mathbf{a}_4)\Delta t/6. \tag{8}$$


- *A. Sagristà, S. Jordan, and F. Sadlo are with Heidelberg University, Germany. A. Just is with ZAH at Heidelberg University, Germany. E-mail: toni.sagrista@iwr.uni-heidelberg.de, {jordan, just}@ari.uni-heidelberg.de, sadlo@uni-heidelberg.de.*
- *F. Dias and L. G. Nonato are with Universidade de São Paulo, São Carlos, Brazil. E-mail: {fabio, gnonato}@icmc.usp.br.*